MA261 Quiz 8

July 22, 2016

Problem 1.

Given a vector field $\mathbf{F} = \langle 2x - y, 3y^2 - x \rangle$, evaluate the line integral $\int_{\mathbf{C}} \mathbf{F} \cdot d\mathbf{r}$ where C is given by

$$\mathbf{r}(t) = \langle t^2 + 1, 1 - 2t \rangle,\ 0 \leq t \leq 1$$

Solution.

Since $\frac{\partial \mathbf{F}_x}{\partial y} = \frac{\partial \mathbf{F}_y}{\partial x} = -1$, $\mathbf{F}$ is a conservative vector field. We can find a function $f(x, y)$ such that $\nabla f = \mathbf{F}$ and use the fundamental theorem for line integrals. A quick computation gives

$$f(x, y) = x^2 + y^3 - xy + C$$

Therefore, $\int_{\mathbf{C}} \mathbf{F} \cdot d\mathbf{r} = f(2, -1) - f(1, 1) = 4$

Problem 2.

Evaluate the line integral along $\int_{\mathbf{C}} x \cos z \, ds$ along C where C is the circular helix given by the equations

$$\begin{cases} x &= 2\cos t \\ y &= 2\sin t \qquad (0 \leq t \leq 4\pi) \\ z &= t \end{cases}$$

Solution.

$$ds = \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dy})^2 + (\frac{dz}{dt})^2}\, dt = \sqrt{5}dt$$

$$\int_{\mathbf{C}} x \cos z \, ds = \int_0^{4\pi} 2\sqrt{5} \cos^2 t \, dt = 4\pi\sqrt{5}$$